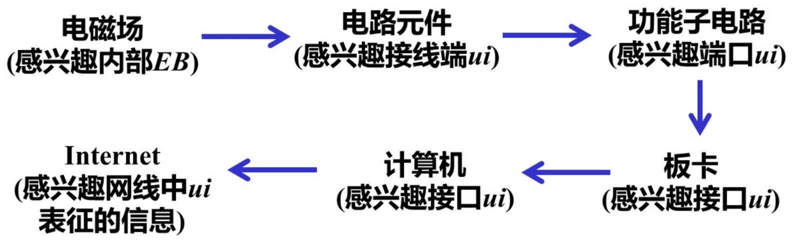
第10章 双口网络

- ## Why Two-port?

  抽象的力量

  对等效的理解

  二端口的分析手段：KCL+KVL+元件约束

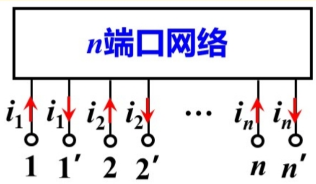
- 10.1 双口网络概述
  - **1. 端口（port）和端口条件**

    电流从一对端纽的一端子流入，从另一端子**全部**流出时，称其为**一个端口**，而组成端口的端子之间的电流关系被称为**端口条件**。

    数学表达式为 $i_m = i_k (m \neq k)$ ，则m与k构成一个端口。

    2. n端口网络

    $2n$端网络，两两成对满足端口条件，便构成n端口网络。

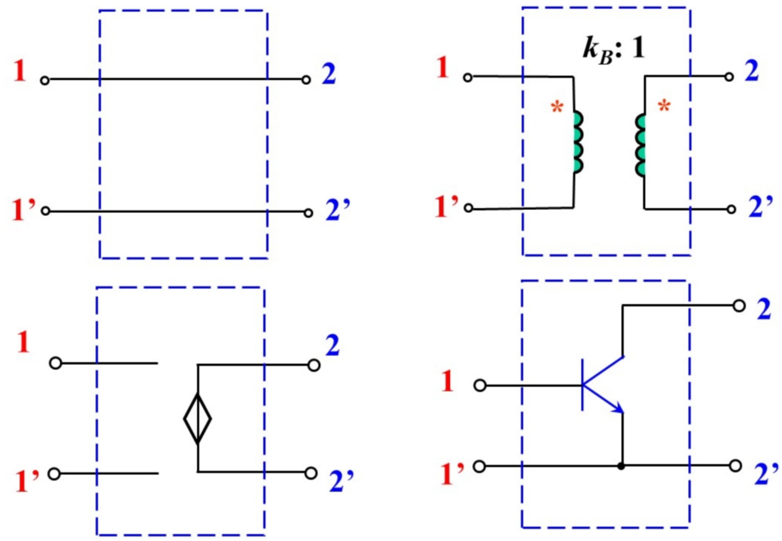
    3.二端口网络（two-port） Franz Breisig 1920提出

    一个满足端口条件的四端网络。它与外部电路通过两个端口连接，并把1-1′端称为双口网络的输入端口（input port），即入口；把2-2′端称为双口网络的输出端口（output port），即出口。

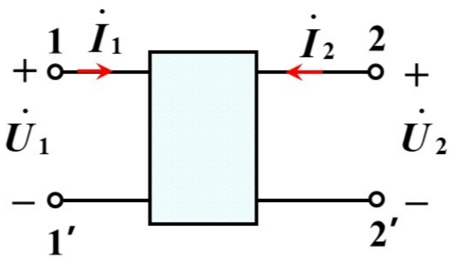
    

    注意 **参考方向**：u上+下-，i从u的+端流入。

双口网络举例

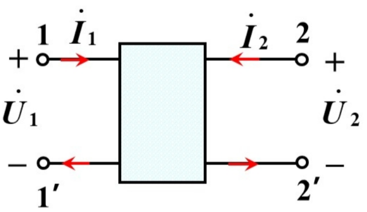


- 10.2 双口网络的方程及参数
  - **1.双口网络方程**



端口物理量4个 $\dot{U}_1$ $\dot{U}_2$ $\dot{I}_1$ $\dot{I}_2$

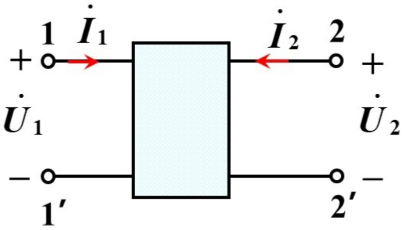
如何描述二端口网络的电压电流关系?



端口物理量4个 $\dot{U}_1$ $\dot{U}_2$ $\dot{I}_1$ $\dot{I}_2$

用	来表示
$\dot{U}_1$ $\dot{U}_2$	$\dot{I}_1$ $\dot{I}_2$
$\dot{I}_1$ $\dot{I}_2$	$\dot{U}_1$ $\dot{U}_2$
$\dot{I}_1$ $\dot{U}_2$	$\dot{U}_1$ $\dot{I}_2$
$\dot{U}_1$ $\dot{I}_2$	$\dot{I}_1$ $\dot{U}_2$
$\dot{I}_2$ $\dot{U}_2$	$\dot{I}_1$ $\dot{U}_1$
$\dot{I}_1$ $\dot{U}_1$	$\dot{I}_2$ $\dot{U}_2$

1）导纳参数方程(Y参数方程)



$$\begin{cases} \dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2 \end{cases}$$

矩阵形式

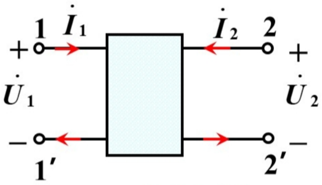
$$\begin{bmatrix} \dot I_1 \\ \dot I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot U_1 \\ \dot U_2 \end{bmatrix}$$

令

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

向量形式

$$\dot I = Y\dot U$$



对于某一黑箱二端口，如何获得Y参数?

此处可以有弹幕

类比一端口网络端口导纳的求法

加压求流

Y参数的**实验测定**

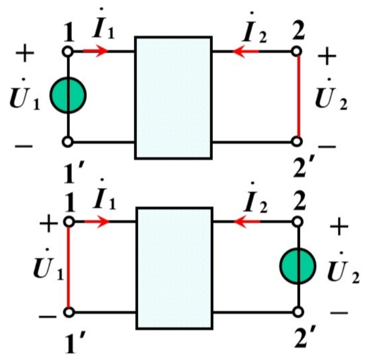
$$Y_{11} = \left.\frac{\dot I_1}{\dot U_1}\right|_{\dot U_2=0}$$ **输入导纳**

$$Y_{21} = \left.\frac{\dot I_2}{\dot U_1}\right|_{\dot U_2=0}$$ **转移导纳**

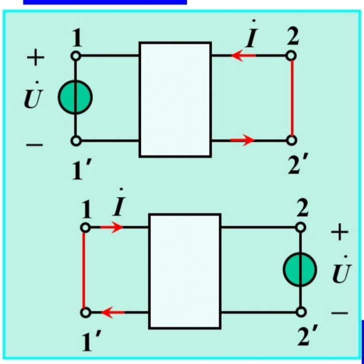
$$Y_{12} = \left.\frac{\dot I_1}{\dot U_2}\right|_{\dot U_1=0}$$ **转移导纳**

$$Y_{22} = \left.\frac{\dot I_2}{\dot U_2}\right|_{\dot U_1=0}$$ **输出导纳**

称Y为**短路导纳参数矩阵**



互易二端口 激励无论加在哪侧，**对侧**产生的响应都一样



$$\begin{bmatrix} \dot I_1 \\ \dot I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot U_1 \\ \dot U_2 \end{bmatrix}$$

$\dot I = Y_{21}\dot U$

$\dot I = Y_{12}\dot U$

$Y_{12} = Y_{21}$

互易二端口网络**四个参数中只有三个是独立的**

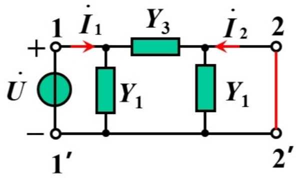
由线性电阻组成的二端口

互易定理

互易二端口

对称二端口 两个端口外特性(输入端/输出端)完全一样

$$\begin{bmatrix}\dot I_1\\ \dot I_2\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\dot U_1\\ \dot U_2\end{bmatrix}$$



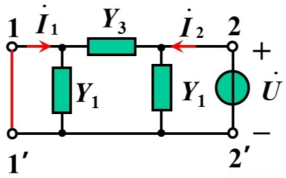
$\dot I_2=Y_{21}\dot U$

$\dot I_1=Y_{11}\dot U$

$\dot I_2=Y_{22}\dot U$

$\dot I_1=Y_{12}\dot U$

$Y_{11}=Y_{22}$ $Y_{12}=Y_{21}$



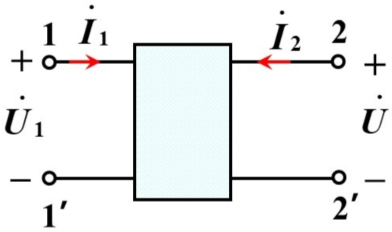
对称二端口只有**两个参数是独立**的。

结构对称

结构对称的二端口 → 对称二端口(电气对称)

对称二端口 ⊗→ 结构对称的二端口

2）阻抗参数方程(*Z*参数方程)



$$\begin{cases}\dot U_1=Z_{11}\dot I_1+Z_{12}\dot I_2\\ \dot U_2=Z_{21}\dot I_1+Z_{22}\dot I_2\end{cases}$$

矩阵形式

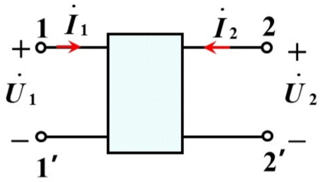
$$\begin{bmatrix}\dot U_1\\ \dot U_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\end{bmatrix}$$

令

$$Z=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}$$

相量形式

$$\dot U=Z\dot I$$



类比一端口网络端口阻抗的求法

加流求压

*Z*参数的**实验测定**

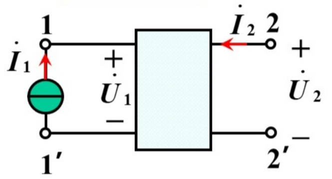
$Z_{11}=\dfrac{\dot U_1}{\dot I_1}\Big|_{\dot I_2=0}$ **输入阻抗**

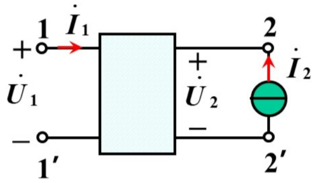
$Z_{21}=\dfrac{\dot U_2}{\dot I_1}\Big|_{\dot I_2=0}$ **转移阻抗**

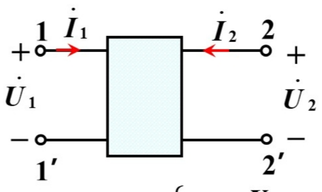
$Z_{12}=\dfrac{\dot U_1}{\dot I_2}\Big|_{\dot I_1=0}$ **转移阻抗**

$Z_{22}=\dfrac{\dot U_2}{\dot I_2}\Big|_{\dot I_1=0}$ **输出阻抗**

称*Z*为**开路阻抗参数矩阵**







由Y参数方程

$$\begin{cases}\dot{I}_1=Y_{11}\dot{U}_1+Y_{12}\dot{U}_2\\ \dot{I}_2=Y_{21}\dot{U}_1+Y_{22}\dot{U}_2\end{cases}\xrightarrow{\text{解出}}\dot{U}_1,\dot{U}_2$$

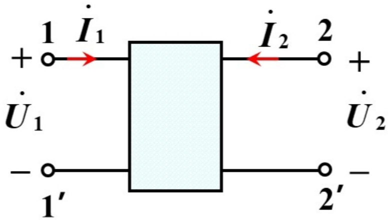
即
$$\begin{cases}\dot{U}_1=\dfrac{Y_{22}}{\Delta}\dot{I}_1+\dfrac{-Y_{12}}{\Delta}\dot{I}_2=Z_{11}\dot{I}_1+Z_{12}\dot{I}_2\\ \dot{U}_2=\dfrac{-Y_{21}}{\Delta}\dot{I}_1+\dfrac{Y_{11}}{\Delta}\dot{I}_2=Z_{21}\dot{I}_1+Z_{22}\dot{I}_2\end{cases}$$

其中 $\Delta=Y_{11}Y_{22}-Y_{12}Y_{21}\neq 0$ 　　前提：Y非奇异

互易二端口 　$Y_{12}=Y_{21}\rightarrow \boxed{Z_{12}=Z_{21}}$

对称二端口 　$Y_{12}=Y_{21}$，$Y_{11}=Y_{22}\rightarrow \boxed{Z_{12}=Z_{21}}$，$\boxed{Z_{11}=Z_{22}}$

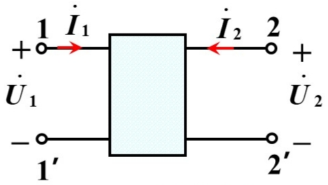
3）混合参数方程(H参数方程)



$$\begin{cases}\dot{U}_1=H_{11}\dot{I}_1+H_{12}\dot{U}_2\\ \dot{I}_2=H_{21}\dot{I}_1+H_{22}\dot{U}_2\end{cases}$$

矩阵形式
$$\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}$$

令
$$H=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}$$

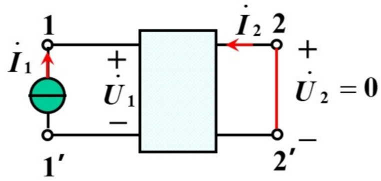


类比一端口网络端口阻抗的求法

H参数的实验测定

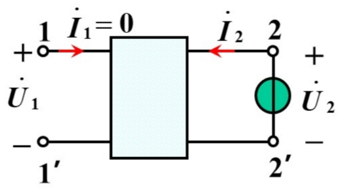
$H_{11}=\left.\dfrac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0}$ 　输入阻抗

$H_{21}=\left.\dfrac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0}$ 　电流比值



$H_{12}=\left.\dfrac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_1=0}$ 　电压比值

$H_{22}=\left.\dfrac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{I}_1=0}$ 　输出导纳



称H为混合参数矩阵

互易二端口 $H_{12} = -H_{21}$

对称二端口 $\Delta H = H_{11}H_{22} - H_{12}H_{21} = H_{11}H_{22} + H_{12}^2 = 1$

4）逆混合参数方程(*G*参数方程)

$$\begin{cases} \dot{I}_1 = G_{11}\dot{U}_1 + G_{12}\dot{I}_2 \\ \dot{U}_2 = G_{21}\dot{U}_1 + G_{22}\dot{I}_2 \end{cases}$$

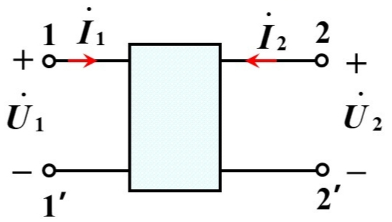
矩阵形式 $$\begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix}$$

令 $$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

G 和 H 互为逆矩阵

$G = H^{-1}$ 或 $H = G^{-1}$

5）传输参数方程(*T*参数方程)



$$\begin{cases} \dot{U}_1 = T_{11}\dot{U}_2 + T_{12}(-\dot{I}_2) \\ \dot{I}_1 = T_{21}\dot{U}_2 + T_{22}(-\dot{I}_2) \end{cases}$$

矩阵形式 $$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

令 $$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \qquad (1)$$

$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \qquad (2)$$

由(2)得

将(3)代入(1)得

$$\begin{cases} \dot{U}_1 = -\dfrac{Y_{22}}{Y_{21}}\dot{U}_2 + \dfrac{1}{Y_{21}}\dot{I}_2 \qquad (3) \\ \dot{I}_1 = \left(Y_{12} - \dfrac{Y_{11}Y_{22}}{Y_{21}}\right)\dot{U}_2 + \dfrac{Y_{11}}{Y_{21}}\dot{I}_2 \end{cases}$$

令

$$T_{11} = -\frac{Y_{22}}{Y_{21}} \quad T_{12} = \frac{-1}{Y_{21}} \quad T_{21} = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} \quad T_{22} = -\frac{Y_{11}}{Y_{21}}$$

即

$$\begin{aligned} \dot{U}_1 &= T_{11}\dot{U}_2 + T_{12}(-\dot{I}_2) \\ \dot{I}_1 &= T_{21}\dot{U}_2 + T_{22}(-\dot{I}_2) \end{aligned}$$

(注意负号)

称为传输参数(T)矩阵

互易二端口 $Y_{12}=Y_{21}$

$$T_{11}=-\frac{Y_{22}}{Y_{21}} \qquad T_{12}=\frac{-1}{Y_{21}}$$

$$T_{21}=\frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}} \qquad T_{22}=-\frac{Y_{11}}{Y_{21}}$$

$T_{11}T_{22}-T_{12}T_{21}$

$$=\frac{\cancel{Y_{11}}Y_{22}}{\cancel{Y_{21}^2}}+\frac{Y_{12}\cancel{Y_{21}}}{\cancel{Y_{21}^2}}-\frac{\cancel{Y_{11}}Y_{22}}{\cancel{Y_{21}^2}}=1$$

对称二端口 $Y_{12}=Y_{21}$，$Y_{11}=Y_{22}$ 则 $T_{11}T_{22}-T_{12}T_{21}=1$，$T_{11}=T_{22}$

T 参数的实验测定（黑箱）

$$\dot{U}_1=T_{11}\dot{U}_2+T_{12}(-\dot{I}_2)$$

$$\dot{I}_1=T_{21}\dot{U}_2+T_{22}(-\dot{I}_2)$$

$$\left.\begin{aligned}T_{11}&=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}\\ T_{21}&=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}\end{aligned}\right\}\text{开路参数}$$

$$\left.\begin{aligned}T_{12}&=\left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}\\ T_{22}&=\left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}\end{aligned}\right\}\text{短路参数}$$

6）反向传输参数方程(T'参数方程)

$$\begin{cases}\dot{U}_2=T'_{11}\dot{U}_1+T'_{12}\dot{I}_1\\ -\dot{I}_2=T'_{21}\dot{U}_1+T'_{22}\dot{I}_1\end{cases}$$

矩阵形式

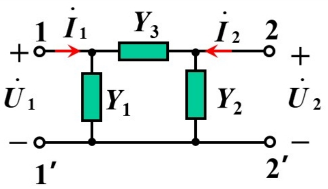
$$\begin{bmatrix}\dot{U}_2\\ -\dot{I}_2\end{bmatrix}=\begin{bmatrix}T'_{11} & T'_{12}\\ T'_{21} & T'_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{I}_1\end{bmatrix}$$

令 $T'=\begin{bmatrix}T'_{11} & T'_{12}\\ T'_{21} & T'_{22}\end{bmatrix}$

T 和 T' 互为逆矩阵

$T=T'^{-1}$ **或** $T'=T^{-1}$

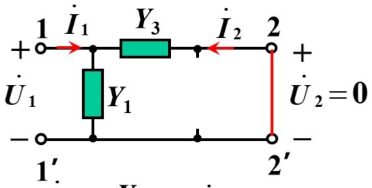
- 例题

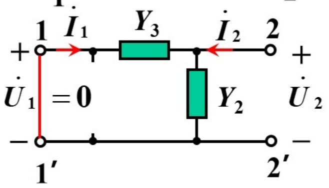


求Y参数。

$$\begin{cases}\dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2\end{cases}$$

解：法1（黑箱思路）





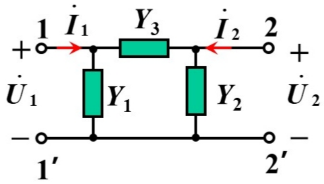
$$Y_{11} = \left.\frac{\dot I_1}{\dot U_1}\right|_{\dot U_2=0} = Y_1 + Y_3$$

$$Y_{21} = \left.\frac{\dot I_2}{\dot U_1}\right|_{\dot U_2=0} = -Y_3$$

$$Y_{12} = \left.\frac{\dot I_1}{\dot U_2}\right|_{\dot U_1=0} = -Y_3$$

$$Y_{22} = \left.\frac{\dot I_2}{\dot U_2}\right|_{\dot U_1=0} = Y_2 + Y_3$$

$$\boxed{Y_{12} = Y_{21} = -Y_3}$$



求Y参数。

$$\begin{cases}\dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2\end{cases}$$

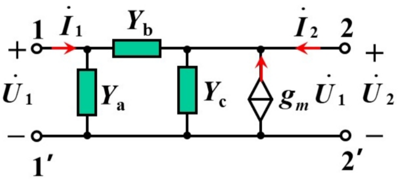
解：法2，对电路二端口，可直接求端口电压电流关系

$$\begin{cases}\dot I_1 = Y_1\dot U_1 + Y_3(\dot U_1 - \dot U_2) \\ \dot I_2 = Y_2\dot U_2 + Y_3(\dot U_2 - \dot U_1)\end{cases}$$

$$Y_{11} = Y_1 + Y_3$$
$$Y_{12} = -Y_3$$
$$Y_{21} = -Y_3$$
$$Y_{22} = Y_2 + Y_3$$

$$Y = \begin{bmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 & Y_2 + Y_3 \end{bmatrix}$$

例题 10.2



求Y参数。

$$\begin{cases}\dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2\end{cases}$$

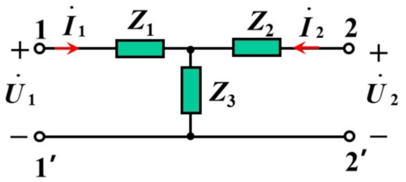
解：对电路二端口，可直接求端口电压电流关系

$$\begin{cases}\dot I_1 = Y_a\dot U_1 + Y_b(\dot U_1 - \dot U_2) \\ \dot I_2 = Y_c\dot U_2 + Y_b(\dot U_2 - \dot U_1) - g_m\dot U_1\end{cases}$$

$$Y_{11} = Y_a + Y_b$$
$$Y_{12} = -Y_b$$
$$Y_{21} = -Y_b - g_m$$
$$Y_{22} = Y_b + Y_c$$

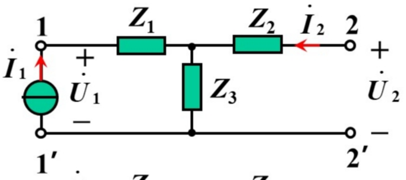
$$Y_{12} \neq Y_{21}$$

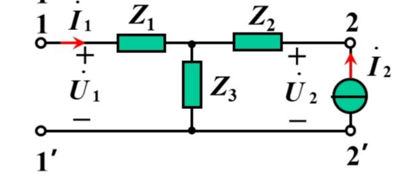
由此可见双口网络内部存在受控源一般不满足互易性。



求Z参数。

$$\begin{bmatrix}\dot U_1\\ \dot U_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\end{bmatrix}$$





解：法1（黑箱测量）

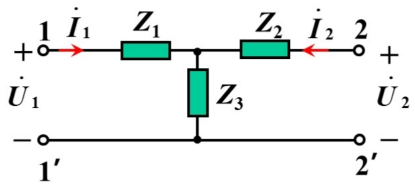
$$Z_{11}=\frac{\dot U_1}{\dot I_1}\bigg|_{\dot I_2=0}=Z_1+Z_3$$

$$Z_{21}=\frac{\dot U_2}{\dot I_1}\bigg|_{\dot I_2=0}=Z_3$$

$$Z_{12}=\frac{\dot U_1}{\dot I_2}\bigg|_{\dot I_1=0}=Z_3$$

$$Z_{22}=\frac{\dot U_2}{\dot I_2}\bigg|_{\dot I_1=0}=Z_2+Z_3$$

$$\boxed{Z_{12}=Z_{21}=Z_3}$$



求Z参数。

$$\begin{bmatrix}\dot U_1\\ \dot U_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\end{bmatrix}$$

解：法2，对电路二端口，可直接求端口电压电流关系

$$\begin{cases}\dot U_1=Z_1\dot I_1+Z_3(\dot I_1+\dot I_2)\\ \dot U_2=Z_2\dot I_2+Z_3(\dot I_2+\dot I_1)\end{cases}$$

$$Z_{11}=Z_1+Z_3$$

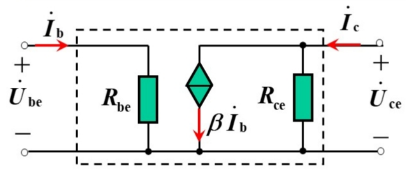
$$Z_{12}=Z_3$$

$$Z_{21}=Z_3$$

$$Z_{22}=Z_2+Z_3$$

$$Z=\begin{bmatrix}Z_1+Z_3 & Z_3\\ Z_3 & Z_2+Z_3\end{bmatrix}$$

互易二端口



求H参数。

$$\begin{bmatrix}\dot U_1\\ \dot I_2\end{bmatrix}=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot U_2\end{bmatrix}$$

解：法1（直接列写）

$$\dot U_{be}=R_{be}\dot I_b$$

$$\dot I_c=\beta\dot I_b+\frac{\dot U_{ce}}{R_{ce}}$$

$$H_{11}=R_{be}\qquad H_{12}=0$$

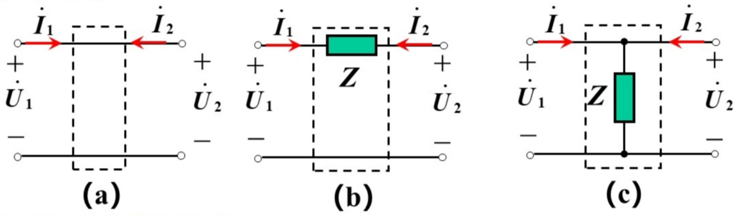
$$H_{21}=\beta\qquad H_{22}=\frac{1}{R_{ce}}$$

$$H=\begin{bmatrix}R_{be} & 0\\ \beta & \dfrac{1}{R_{ce}}\end{bmatrix}$$

$$\boxed{H_{12}\neq -H_{21}}$$

因为含有受控源，所以非互易

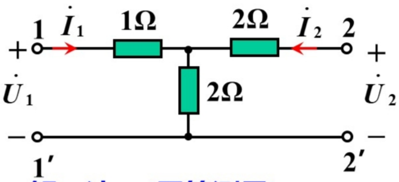
求T参数。



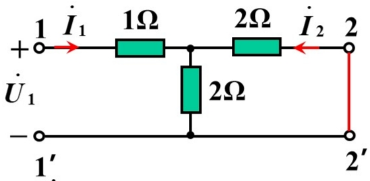
解：法1（直接列写）

(a) $\begin{cases}\dot{U}_1=\dot{U}_2\\ \dot{I}_1=-\dot{I}_2\end{cases}$ (b) $\begin{cases}\dot{U}_1=\dot{U}_2-Z\dot{I}_2\\ \dot{I}_1=-\dot{I}_2\end{cases}$ (c) $\begin{cases}\dot{U}_1=\dot{U}_2\\ \dot{I}_1=\dfrac{1}{Z}\dot{U}_2-\dot{I}_2\end{cases}$

$$T=\begin{bmatrix}1&0\\0&1\end{bmatrix}\qquad T=\begin{bmatrix}1&Z\\0&1\end{bmatrix}\qquad T=\begin{bmatrix}1&0\\ \dfrac{1}{Z}&1\end{bmatrix}$$



求T参数



解：法1（黑箱测量）

$$T_{11}=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=\frac{1+2}{2}=1.5$$

$$T_{21}=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=0.5\,\mathrm{S}$$

$$T_{12}=\left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}=\frac{\dot{I}_1[1+(2/ /2)]}{0.5\dot{I}_1}=4\,\Omega$$

$$T_{22}=\left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}=\frac{\dot{I}_1}{0.5\dot{I}_1}=2$$

法2　先写出Y或Z参数，再解出T参数

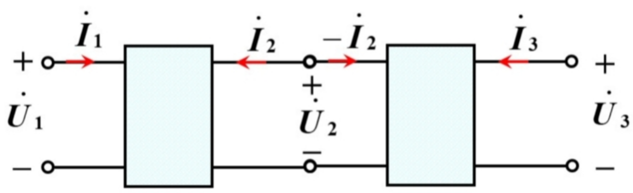
法3（直接列写）根据KCL、KVL列方程并整理

- 10.3 双口网络的互联
  -
    双口网络的作用：

    1) 对子电路进行抽象，便于分析和构造更复杂的电路

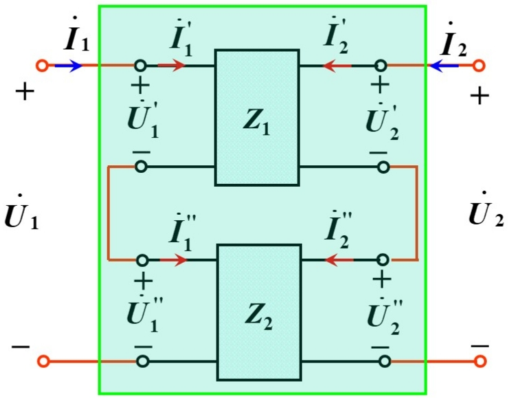
    2) 对现有子电路进行端口描述，便于进行反向工程设计
  - **1.双口网络级联**

    把前一个双口网络的出口与后一个双口网络的入口连接起来，称为双口网络的级联。

  - **2.串-串联**

分别把两个双口网络的入口端相串联，出口端相串联的连接方式，称为**双口网络的串-串联**。



电流满足端口条件

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}=\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}$$

电压满足KVL

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1+\dot{U}''_1\\ \dot{U}'_2+\dot{U}''_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}=\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1+\dot{U}''_1\\ \dot{U}'_2+\dot{U}''_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1\\ \dot{U}'_2\end{bmatrix}+\begin{bmatrix}\dot{U}''_1\\ \dot{U}''_2\end{bmatrix}=Z_1\begin{bmatrix}\dot{I}'_1\\ \dot{I}'_2\end{bmatrix}+Z_2\begin{bmatrix}\dot{I}''_1\\ \dot{I}''_2\end{bmatrix}$$

整理得 $$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\{Z_1+Z_2\}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=Z\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}$$

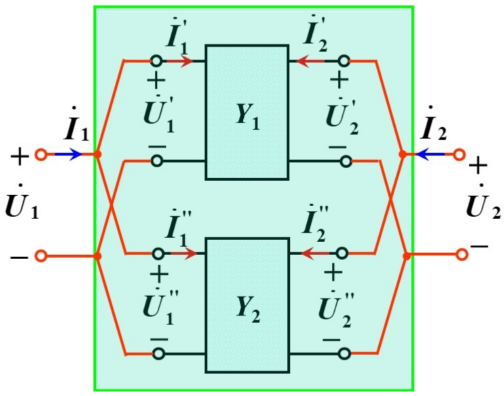
所以有 $Z=Z_1+Z_2$

如果有n个双口网络串联，则有

$$Z=Z_1+Z_2+\cdots+Z_n$$

- 3.并-并联

分别把两个双口网络的入口端相并联，出口端相并联的连接方式，称为**双口网络的并-并联**。



电流满足KCL

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}'_1+\dot{I}''_1\\ \dot{I}'_2+\dot{I}''_2\end{bmatrix}$$

电压满足端口条件

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}'_1\\ \dot{U}'_2\end{bmatrix}=\begin{bmatrix}\dot{U}''_1\\ \dot{U}''_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}\dot{U}_1'\\ \dot{U}_2'\end{bmatrix}=\begin{bmatrix}\dot{U}_1''\\ \dot{U}_2''\end{bmatrix}$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}_1'+\dot{I}_1''\\ \dot{I}_2'+\dot{I}_2''\end{bmatrix}=\begin{bmatrix}\dot{I}_1'\\ \dot{I}_2'\end{bmatrix}+\begin{bmatrix}\dot{I}_1''\\ \dot{I}_2''\end{bmatrix}=Y_1\begin{bmatrix}\dot{U}_1'\\ \dot{U}_2'\end{bmatrix}+Y_2\begin{bmatrix}\dot{U}_1''\\ \dot{U}_2''\end{bmatrix}$$

整理得 $\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\{Y_1+Y_2\}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=Y\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$

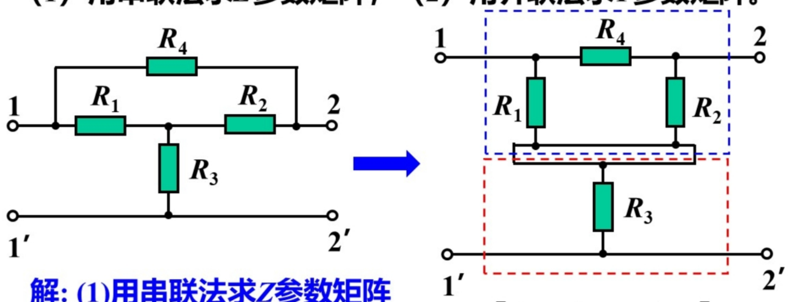
所以有 $Y=Y_1+Y_2$

如果有n个双口网络并联，则有

$$Y=Y_1+Y_2+\cdots+Y_n$$

- **例题**

（1）用串联法求Z参数矩阵；（2）用并联法求Y参数矩阵。

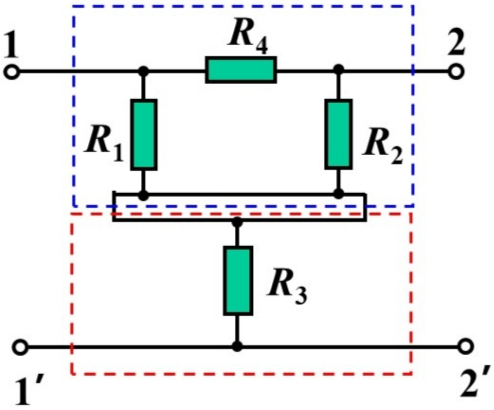


解: (1)用串联法求Z参数矩阵

$$Y_1=\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_4} & -\frac{1}{R_4}\\ -\frac{1}{R_4} & \frac{1}{R_2}+\frac{1}{R_4}\end{bmatrix}$$

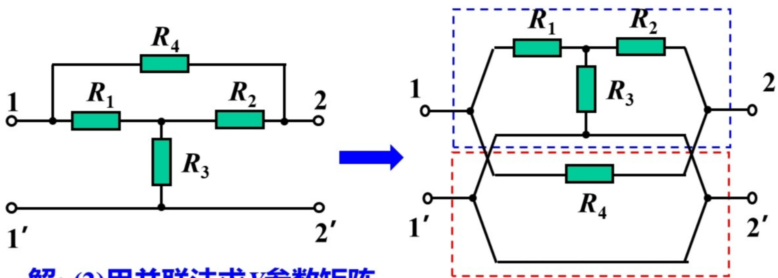
$$Z_1=Y_1^{-1}=\frac{1}{\Delta Y_1}\begin{bmatrix}\frac{1}{R_2}+\frac{1}{R_4} & \frac{1}{R_4}\\ \frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_4}\end{bmatrix}$$

$$\Delta Y_1=\frac{R_1+R_2+R_4}{R_1R_2R_4}$$



$$Z_2=\begin{bmatrix} R_3 & R_3 \\ R_3 & R_3 \end{bmatrix}$$

$$Z_1+Z_2=\begin{bmatrix} R_3+\dfrac{R_1R_4+R_1R_2}{R_1+R_2+R_4} & R_3+\dfrac{R_1R_2}{R_1+R_2+R_4} \\ R_3+\dfrac{R_1R_2}{R_1+R_2+R_4} & R_3+\dfrac{R_2R_4+R_1R_2}{R_1+R_2+R_4} \end{bmatrix}$$



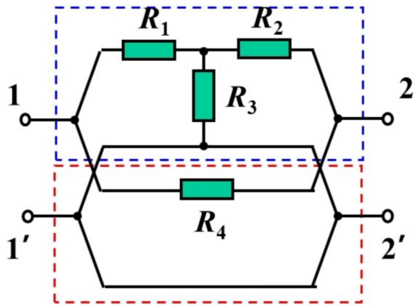
解: (2)用并联法求*Y*参数矩阵

$$[Y]=[Y_1]+[Y_2]$$

$$[Z_1]=\begin{bmatrix} R_1+R_3 & R_3 \\ R_3 & R_2+R_3 \end{bmatrix}$$

$$[Y_1]=[Z_1]^{-1}=\begin{bmatrix} R_1+R_3 & R_3 \\ R_3 & R_2+R_3 \end{bmatrix}^{-1}=\frac{1}{\Delta Z_1}\begin{bmatrix} R_2+R_3 & -R_3 \\ -R_3 & R_1+R_3 \end{bmatrix}$$

$$\Delta Z_1=R_1R_2+R_2R_3+R_1R_3$$

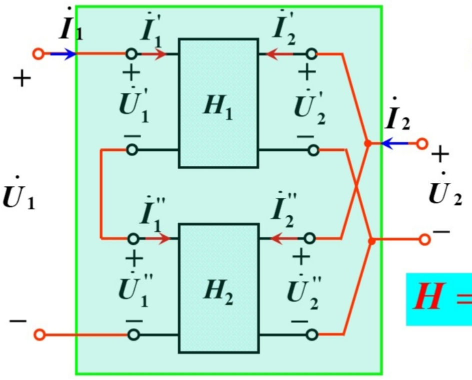


$$[Y_2]=\begin{bmatrix} \dfrac{1}{R_4} & -\dfrac{1}{R_4} \\ -\dfrac{1}{R_4} & \dfrac{1}{R_4} \end{bmatrix}$$

$$[Y]=[Y_1]+[Y_2]=\begin{bmatrix} \dfrac{1}{R_4}+\dfrac{R_2+R_3}{R_1R_2+R_2R_3+R_1R_3} & -\dfrac{1}{R_4}-\dfrac{R_3}{R_1R_2+R_2R_3+R_1R_3} \\ -\dfrac{1}{R_4}-\dfrac{R_3}{R_1R_2+R_2R_3+R_1R_3} & \dfrac{1}{R_4}+\dfrac{R_1+R_3}{R_1R_2+R_2R_3+R_1R_3} \end{bmatrix}$$

- **4.串-并联**

分别把两个双口网络的入口端相串联，出口端相并联的连接方式，称为双口网络的串-并联。



放着前面可以推出

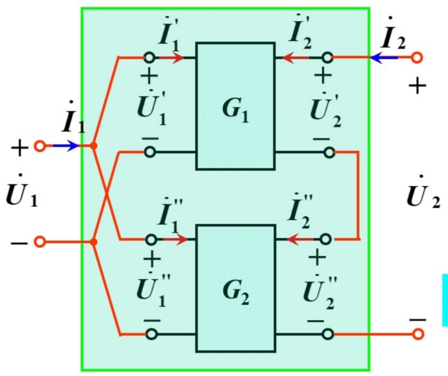
$H=H_1+H_2$

如果有n个双口网络串-并联，则有

$$H=H_1+H_2+\cdots+H_n$$

- **5.并-串联**

分别把两个双口网络的入口端相并联，出口端相串联的连接方式，称为双口网络的并-串联。



放着前面可以推出

$G=G_1+G_2$

如果有n个双口网络并-串联，则有

$$G=G_1+G_2+\cdots+G_n$$

- 10.4 双口网络的开路阻抗和短路阻抗
  - **1.开路阻抗**

在双口网络一个端口开路的情况下，从另一个端口所测得的电压与电流的比定义为双口网络的开路阻抗。

1）入口的入端阻抗（出口开路）

$$Z_{OC1}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}=\frac{T_{11}}{T_{21}}$$

$$\dot{U}_1=T_{11}\dot{U}_2+T_{12}(-\dot{I}_2)$$
$$\dot{I}_1=T_{21}\dot{U}_2+T_{22}(-\dot{I}_2)$$

2）出口的入端阻抗（入口开路）

$$Z_{OC2}=\left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}=\frac{T_{22}}{T_{21}}$$

- **2.短路阻抗**

在双口网络一个端口短路的情况下，从另一个端口所测得的电压与电流的比定义为双口网络的短路阻抗。

1）入口的入端阻抗（出口短路）

$$Z_{SC1}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0}=\frac{T_{12}}{T_{22}}$$

$$\dot{U}_1=T_{11}\dot{U}_2+T_{12}(-\dot{I}_2)$$
$$\dot{I}_1=T_{21}\dot{U}_2+T_{22}(-\dot{I}_2)$$

2）出口的入端阻抗（入口短路）

$$Z_{SC2}=\left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{U}_1=0}=\frac{T_{12}}{T_{11}}$$

- **3.开、短路阻抗关系**

$$Z_{OC1}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}=\frac{T_{11}}{T_{21}}$$
$$Z_{OC2}=\left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}=\frac{T_{22}}{T_{21}}$$
$$Z_{SC1}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0}=\frac{T_{12}}{T_{22}}$$
$$Z_{SC2}=\left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{U}_1=0}=\frac{T_{12}}{T_{11}}$$

开路阻抗与短路阻抗存在以下关系

$$\frac{Z_{OC1}}{Z_{SC1}}=\frac{Z_{OC2}}{Z_{SC2}}=\frac{T_{11}T_{22}}{T_{21}T_{21}}$$

开路阻抗与短路阻抗中仅有三个是独立的。

只须测量三个值即可。求解T参数时，缺少的方程，可补充

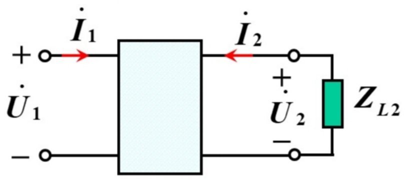
$$T_{11}T_{22}-T_{12}T_{21}=1$$

若无源双口网络对称，由T_{11}=T_{22}又得到

$$\begin{cases}Z_{OC1}=Z_{OC2}=Z_{OC}\\Z_{SC1}=Z_{SC2}=Z_{SC}\end{cases}$$

此时只有开、短路阻抗是独立的，在任意一个端口处测量即可。

- 10.5 对称双口网络的特性阻抗
  - **1.入口的入端阻抗**



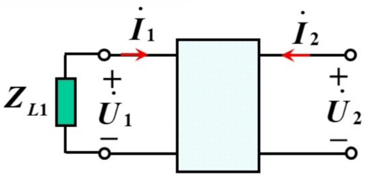
在出口处接入负载Z_{L2}，则入口的入端阻抗

$$\left.\begin{aligned} Z_{in} &= \frac{\dot{U}_1}{\dot{I}_1} = \frac{T_{11}\dot{U}_2 - T_{12}\dot{I}_2}{T_{21}\dot{U}_2 - T_{22}\dot{I}_2} \\ \dot{U}_2 &= -Z_{L2}\dot{I}_2 \end{aligned}\right\} \Rightarrow Z_{in} = \frac{-T_{11}Z_{L2}\dot{I}_2 - T_{12}\dot{I}_2}{-T_{21}Z_{L2}\dot{I}_2 - T_{22}\dot{I}_2} = \frac{T_{11}Z_{L2} + T_{12}}{T_{21}Z_{L2} + T_{22}}$$

将入口端阻抗转变为Z_{in}，**有阻抗变换作用**

但一般情况下，$Z_{in} \neq Z_{L2}$。

- 2.出口的入端阻抗



在入口处接入负载Z_{L1}，则出口的入端阻抗

$$\left.\begin{aligned} Z_{out} &= \frac{\dot{U}_2}{\dot{I}_2} = \frac{T_{22}\dot{U}_1 - T_{12}\dot{I}_1}{T_{21}\dot{U}_1 - T_{11}\dot{I}_1} \\ \dot{U}_1 &= -Z_{L1}\dot{I}_1 \end{aligned}\right\} \Rightarrow Z_{out} = \frac{-T_{22}Z_{L1}\dot{I}_1 - T_{12}\dot{I}_1}{-T_{21}Z_{L1}\dot{I}_1 - T_{11}\dot{I}_1} = \frac{T_{22}Z_{L1} + T_{12}}{T_{21}Z_{L1} + T_{11}}$$

尽管取$Z_{L1} = Z_{L2}$，但一般情况下，$Z_{in} \neq Z_{out}$。

现在考虑**对称双口网络**的，特殊情况。

显然，若取$Z_{L1} = Z_{L2}$，则必然有

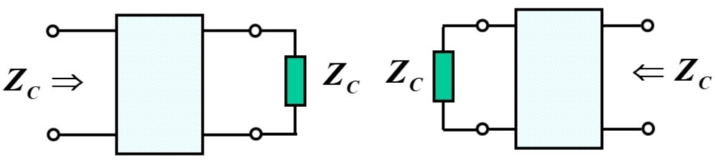
$$\boxed{Z_{in} = Z_{out} = \frac{\dot{U}_2}{\dot{I}_2} = \frac{T_{11}Z_{L1} + T_{12}}{T_{21}Z_{L1} + T_{11}}}$$

进一步，令$Z_{in} = Z_{out} = Z_{L1} = Z_{L2} = Z_C$，就有

$$Z_C = \frac{T_{11}Z_C + T_{12}}{T_{21}Z_C + T_{11}}$$

于是，解方程可得

$$\boxed{Z_C = \sqrt{\frac{T_{12}}{T_{21}}}}$$ **对称双口网络的特性阻抗**



特点:

(1) Z_C是双口网络固有性质，又双口网络的参数决定;

(2)负载为Z_C时，对称双口网络的反映阻抗仍为Z_C，体现了阻抗匹配。在功率传输、阻抗匹配及高频天线系统中有广泛的应用。